Hydrostatic Force and Center of Pressure on Plane Surface

Date Performed: [redacted]

Date Submitted: [redacted]

[redacted]

10/14/2008

Mechanical Engineering

Objective:

This lab is intended to experimentally verify the theoretical formulas governing hydrostatic force and pressure center on a submerged planar surface.

Theoretical background:

According to hydrostatics theory, the hydrostatic force on a submerged planar surface is $F = P_{CG} * A$ where $P_{CG} = (P_{atm} + \gamma * h)$ is the hydrostatic pressure at the center of gravity. The center of pressure is found by the formula $y_{CP} = \frac{\gamma_w * I_{xx} * \sin\ \theta}{F}$ (the origin of the coordinate system being at the center of gravity).

The test is based on the principle of static equilibrium: Sum of the moments on a system at static equilibrium is equal to zero. Also, the set up relies on the fact that the hydrostatic force on the circumference of a torroid passes through its center to cancel the moments of unwanted hydrostatic forces (on the circumference).

Equipment:

- ✓ Set of masses
- ✓ Water supply (hose)
- ✓ Water tank with draining tap
- ✓ Torroid mounted on pivoting arm (with mass holder and balancing system)
- ✓ Water height measurement device
- ✓ Ruler

Procedure:

Initial setup:

The immersed quarter torroid is balanced using the mass m (as seen on the schematic below).

Procedure:

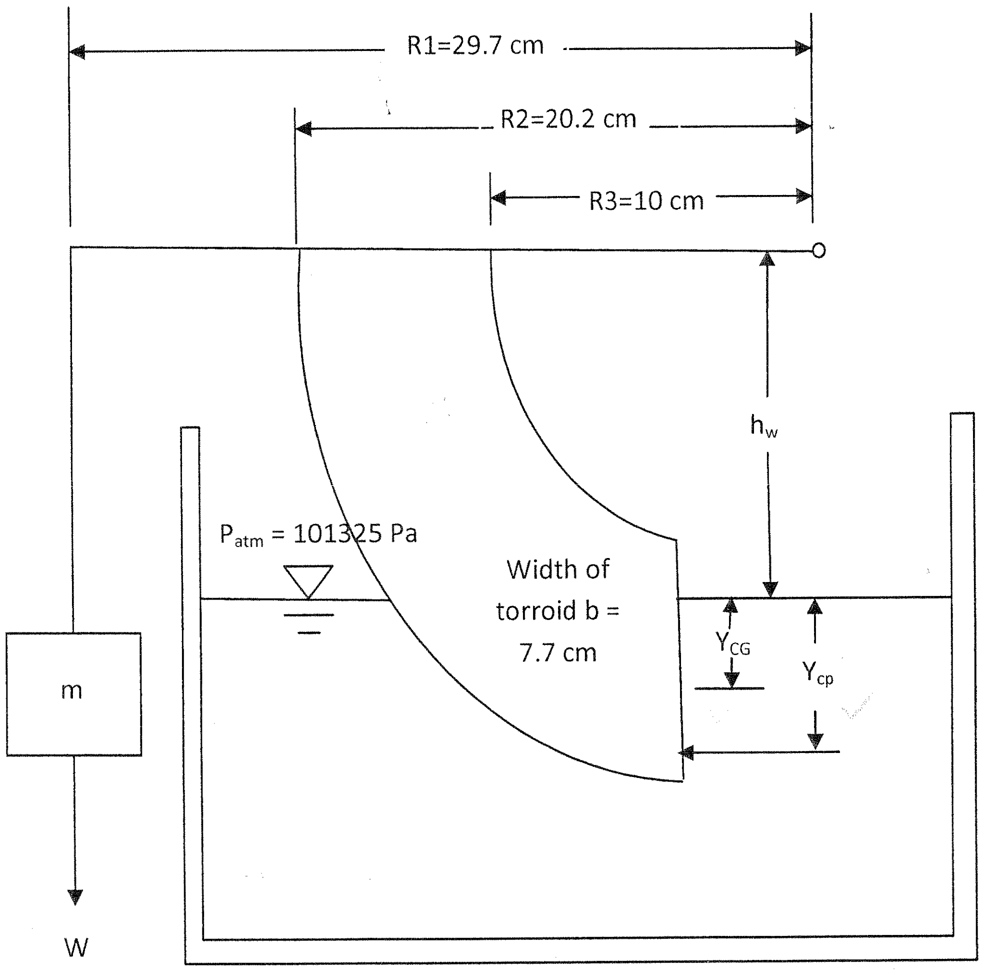
[redacted]

[redacted]

10/14/2008

Mechanical Engineering

Setup



[redacted]

10/14/2008

Mechanical Engineering

Collected Data table:

		Iterations									
Independent	Mass (kg)	211	190	170	150	130	110	90	60	30	5
Dependent	Water Level h_w(mm)	150 (REF)	144.0	138.4	132.5	126.6	119.8	112.8	100.8	85.4	64.8

Data Processing:

Formulas used

Moment due to weight:

$$M_w = m * g * R1$$

$$y_{CG} = \frac{\left[(R_2 - R_3) - (h_{w_{ref}} - h_w)\right]}{2}$$

Submerged Area:

$$A_{sub} = 2 * \ b * y_{CG}$$

Center of pressure:

$$y_{CP} = y_{CG} + \frac{\gamma_w * I_{xx} * \sin\ \theta}{F} \quad with\ \ I_{xx} = \frac{b*(2*y_{CG})^3}{12} \text{ And } \theta = 90^\circ$$

Theoretical pressure force

$$F = (P_{atm} + \gamma_w * y_{CG}) * A_{sub}$$

$$F_{effective} = \gamma_w * y_{CG} * A_{sub}$$

Calculated moment due to theoretical pressure force:

$$M_f = F_{effective} * \left[\left(R_3 + y_{CP} + h_{w_{ref}} - h_w\right)\right]$$

Note: The atmospheric pressure was not included in the calculation of $F_{effective}$ because the moment effect of the atmospheric pressure is balanced out over the top and bottom-right surfaces of the torroid. Consequently, the force $F_{effective}$ measured through the weights is only due to hydrostatic pressure.

Mechanical Engineering

However, the center of pressure is still affected by the effect of the atmospheric pressure which still contributes to the value of pressure at the center of gravity of the submerged surface.

Calculations: Using Excel

Error propagation on M_f

$$\Delta y_{CG} = \frac{1}{2} * \sqrt{\Delta R_2{}^2 + \Delta R_3{}^2 + \Delta h_{w_{ref}}{}^2 + \Delta h_w{}^2}$$

Considering that R_1 R_2 and R_3 and b were measured using a straight edge marked at every millimeters, our eye error on the measurement is about 0.5 mm.

Considering that the measurements on the water level were taken with a vernier scale with a precision of 0.1 mm, the error is about 0.1 mm.

Note: We did not factor in the error from identifying the horizontal position of the pivot arm with the bubble level.

It follows that $\Delta y_{CG} = 3.60555 * 10^{-4}\ m$

$$\Delta A_{sub} = 2\sqrt{(y_{CG} * \Delta b)^2 + (b * \Delta y_{CG})^2}$$

$$\Delta F_{effective} = \gamma_W \sqrt{(A_{sub} * \Delta y_{CG})^2 + (y_{CG} * \Delta A_{sub})^2}$$

Let $h_{CP} = \left(R_3 + y_{CP} + h_{w_{ref}} - h_w\right)$

Then $\Delta h_{CP} = \sqrt{\Delta R_3{}^2 + \Delta y_{CG}{}^2 + \Delta h_{w_{ref}}{}^2 + \Delta h_w{}^2}$

$$\Delta h_{CP} = 6.324555 * 10^{-4}\ m$$

$$\Delta M_f = \sqrt{(h_{cp} * \Delta F)^2 + (F * \Delta h_{CP})^2}$$

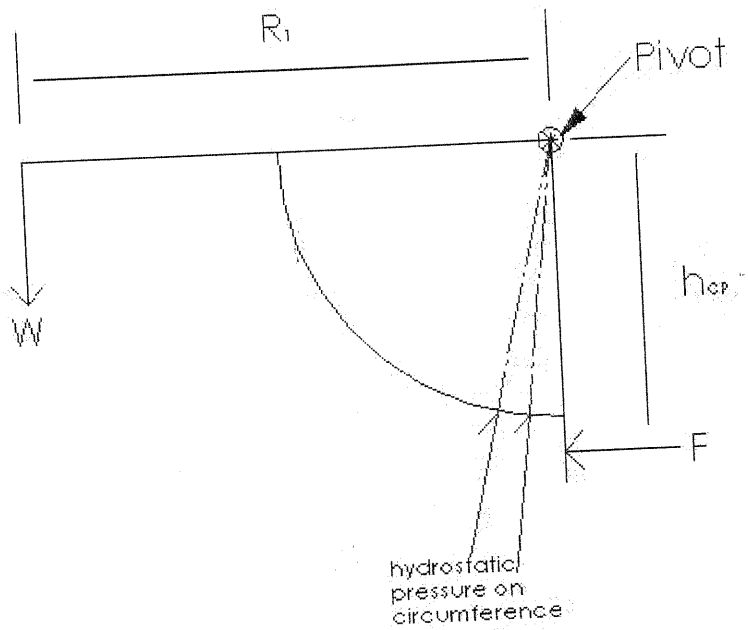
Error propagation on M_w

$W = mg$ is considered an exact value

Therefore, $\Delta M_w = m * g\ \Delta R_1$

10/14/2008

Mechanical Engineering



Discussions

We found that there is effectively hydrostatic force acting on the immersed torroid surface. The slope of the plot M_f vs. M_w is 0.9802 (almost 1 considering the measurement errors) suggesting that the moment from the weight was effectively balanced by the moment of the expected hydrostatic pressure force. The scatter R^2 0.9976 is very close to 1, meaning that M_w and M_f are in an almost perfect linear relationship. The circular shape of the torroid is advantageous in that it directs the elementary pressure forces on the circumference toward the center. Since the center of the torroid is also the pivot point, the moment of those elementary pressure forces is conveniently zero.

The relative error on M_w ($\Delta Mw/Mw$) is constant because the fraction reduces in the following way:

$$M_w = \frac{m * g \; \Delta R_1}{m * g * R1} = \frac{\Delta R_1}{R1}$$

Also, the relative error on the moment from the weight is 0.168%. This value is very satisfactory for an experimental quantity. The relative error on the calculated moment of the pressure force is less than 2.824% (except 6.114% which is still less than 10%) meaning that the measurements of water level were accurate enough for the structure of the hydrostatic formulas (the error on measurements kept the relative error low enough). In addition, the relative error between the weight moment and the theoretical moment is less than 5.46% for most of the data (except for two values which deviated largely from the 8 others). This result supports the fact that hydrostatic pressure is effectively the force balancing the weights. Overall, the setup for the experiment was good at verifying the theories in question.

Set-up Data						
R3 (m)	0.1	Density (kg/m^3)		997.0089	ΔR3 (m)	0.0005
R2 (m)	0.202	g (m/s^2)		9.80665	ΔR2 (m)	0.0005
R1 (m)	0.297	γw (N/m^3)		9777.317	ΔR1 (m)	0.0005
b (m)	0.077				Δb (m)	0.0005
P_{atm} (Pa)	101325				Δh_w (m)	0.0001
T (°C)	24				Δy_{CG} (m)	3.61E-04

Δh_{CP} (m)	6.32E-04

Mass (kg)	0.211	0.19	0.17	0.15	0.13	0.11	0.09	0.06	0.03	0.005
h_w (m)	0.15	0.144	0.1384	0.1325	0.1266	0.1198	0.1128	0.1008	0.0854	0.0648
M_w (N.m)	0.614553336	0.553389	0.495138	0.436886	0.378635	0.320383	0.262132	0.174755	0.087377	0.014563
y_{CG} (m)	0.051	0.048	0.0452	0.04225	0.0393	0.0359	0.0324	0.0264	0.0187	0.0084
A_{sub} (m^2)	0.007854	0.007392	0.006961	0.006507	0.006052	0.005529	0.00499	0.004066	0.00288	0.001294
I_{xx} (m^4)	6.80942E-06	5.68E-06	4.74E-06	3.87E-06	3.12E-06	2.38E-06	1.75E-06	9.45E-07	3.36E-07	3.04E-08
F (N)	799.7228936	752.4635	708.3793	661.9589	615.5647	562.126	507.1519	412.9963	292.3223	131.1803
F_{eff} (N)	3.916343565	3.469149	3.076219	2.687781	2.325549	1.94057	1.580631	1.049417	0.526531	0.106243
y_{CP} (m)	0.051083251	0.048074	0.045265	0.042307	0.039349	0.035941	0.032434	0.026422	0.018711	0.008402
M_f (N.m)	0.591693918	0.534505	0.482552	0.429527	0.378482	0.322409	0.268128	0.184301	0.096519	0.020569
ΔA_{sub} (m)	7.54E-05	7.34E-05	7.16E-05	6.98E-05	6.80E-05	6.61E-05	6.43E-05	6.15E-05	5.86E-05	5.62E-05
ΔF_{eff} (m)	4.67E-02	4.32E-02	4.00E-02	3.68E-02	3.37E-02	3.03E-02	2.69E-02	2.14E-02	1.48E-02	6.49E-03
h_{CP} (m)	1.51E-01	1.54E-01	1.57E-01	1.60E-01	1.63E-01	1.66E-01	1.70E-01	1.76E-01	1.83E-01	1.94E-01
ΔM_f (N.m)	0.007476232	0.007007	0.006576	0.006127	0.005685	0.005183	0.004673	0.003814	0.002726	0.001258
ΔM_w (N.m)	0.001034602	0.000932	0.000834	0.000735	0.000637	0.000539	0.000441	0.000294	0.000147	2.45E-05
$\Delta M_f/M_f$	1.264%	1.311%	1.363%	1.426%	1.502%	1.607%	1.743%	2.069%	2.824%	6.114%
$\Delta M_w/M_w$	0.168%	0.168%	0.168%	0.168%	0.168%	0.168%	0.168%	0.168%	0.168%	0.168%
(Mf-Mw)/Mw	-3.72%	-3.41%	-2.54%	-1.68%	-0.04%	0.63%	2.29%	5.46%	10.46%	41.24%

[redacted] [redacted]

[redacted] [redacted]